QUESTIONS TO BE ANSWERED

Some interesting problems arise; will the drag coefficients also coincide for forced and natural convection when plotted against Reynolds or modified Grashof numbers just as the Nusselt numbers do?

Should the velocity $[g(\beta\Delta t)L]^{1/2}$ be added vectorially to the velocity V to arrive at an appropriate velocity for use in a Reynolds number that will predict the heat transfer coefficient with mixed forced and natural convection flow?

NOTATION

c_p = heat capacity of fluid, $Wkg^{-1}K^{-1}$
Fr = Froude number, $V/(gL)^{1/2}$ dimensionless
g = acceleration of gravity, ms^{-2}
Gr = Grashof number, dimensionless
Gr' = modified Grashof number $Gr^{1/2}$, dimensionless
$\bar{h}$ = circumferentially averaged surface coefficient of heat transfer, $Wm^{-2}K^{-1}$
k = thermal conductivity of fluid, $Wm^{-1}K^{-1}$
L = characteristic length, m
$\overline{Nu}$ = average Nusselt number, $\bar{h}L/k$, dimensionless
Pr = Prandtl number, $c_p\mu/k$, dimensionless
Re = Reynolds number, LV/ν, dimensionless
t = temperature, K
Δt = temperature difference, $t_s - t_\infty$, K
V = velocity, ms^{-1}

Greek Letters

β = temperature coefficient of thermal expansion, K^{-1}
ν = kinematic viscosity, m^2s^{-1}
μ = dynamic viscosity, $kgm^{-1}s^{-1}$

Subscripts

f = evaluated at the film temperature $\frac{1}{2}(t_s + t_\infty)$
s = at surface
∞ = of unheated fluid

LITERATURE CITED

Hilpert, R., "Wärmeaufgabe von geheizten Drähten und Rohren im Lufstrom," *Forsch. Gebiete Ingenierw.*, 4 (5), (1933).
Kreith, Frank, *Principles of Heat Transfer*, 3rd edit., p. 326, Intext Educational Publ., New York (1973a).
———., *ibid.*, p. 390 (1973b).
———., *ibid.*, p. 406 (1973c).
McAdams, W. H., *Heat Transmission*, 3rd edit., p. 176, McGraw-Hill, New York (1954).
Olson, R. M., *Essentials of Engineering Fluid Mechanics*, 3rd edit., p. 268-269, Intext Educational Publ., New York (1973).
White, F. M., *Viscous Fluid Flow*, p. 90, McGraw-Hill, New York (1974).

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Drag Reduction in Film Flow

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It is well known that some dilute polymer solutions in water or other solvents exhibit a reduction of skin friction in turbulent flow. The most common cause mentioned in literature is in terms of extensional flow properties of dilute polymer solutions (Seyer and Metzner, 1969). The resistance of dilute polymer solutions to extensional deformation, commonly termed *elongational viscosity*, is in orders of magnitude higher than that of pure Newtonian solvents (Metzner, 1971). The structure of the flow field in turbulent pipe flow in the wall region consists of counterrotating pairs of eddies. This flow pattern consists, at least in part, of an elongational motion (Gordon et al., 1973).

Now, consider the flow pattern of a wavy film flow. There is an intensive backmixing in the wave trough, qualitatively described first by Kapitza (1948) and mathematically by Massot et al. (1966), as is shown in Figure 1. It is also clear that stretching components of deformation are present when waves appear on the film surface. Although the film flow with its free surface substantially differs from a turbulent pipe flow, there are some similarities in the flow pattern of a film and flow in the wall region of turbulent pipe flow, in particular, in the elements of stretching motion.

The observations stated above form the basis for an explanation of the remarkable reduction in friction coefficient found in experiments on falling films of dilute aqueous Carbopol 934 solutions (Popadić, 1974). For the lowest concentration examined (0.05% w), the friction coefficient was more than 50% lower than predicted by both Equations (2) and (4), as is seen in Figure 2. Equation (2) is obtained for the case of a nonwavy laminar film flow for which

$$f = \frac{2g\delta_0^3}{Q^2} \tag{1}$$

where δ_0 is the film thickness (average thickness for the case of wavy flow) and Q is the volume flow rate per unit width of the plate, or

$$f = \frac{24}{Re} \tag{2}$$

where Re is defined as (Skelland, 1967)

$$Re = \frac{12n}{2n+1}\left(\frac{Q}{g\delta_0}\right)\left(\frac{\rho g\delta_0}{K}\right)^{1/n} \tag{3}$$

In (3) K is the consistency factor and n is the flow behavior index.

For a wavy film flow, an approximative steady periodic solution of a boundary layer form of equation of motion for a power-law liquid has led to (Popadić, 1974)

$$f = 0.833^n \cdot \frac{24}{Re}\Phi(n) \tag{4}$$

where $\Phi(n)$ is a determined function of the flow behavior index.

For Newtonian liquids $n = 1$ and $\Phi(n) = 1$, and Equation (4) gives smaller friction coefficient for a wavy flow than for the nonwavy laminar flow.

A very good agreement between predicted and experi-

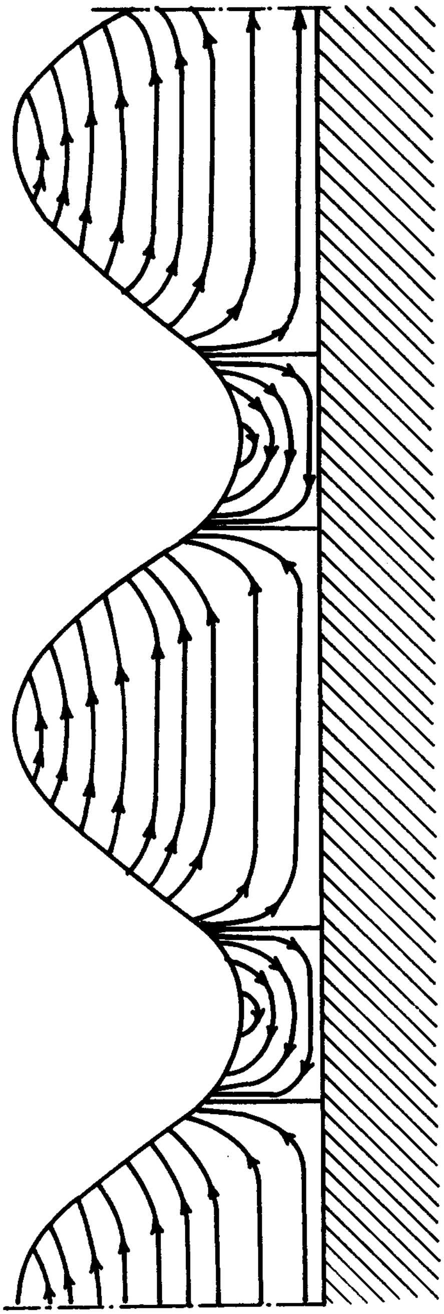

Fig. 1. Instantaneous streamlines of a falling liquid film (Massot et al., 1966).

mental values for Newtonian liquids is seen in Figure 3. For dilute aqueous Carbopol solutions, drag is reduced significantly, but the effect disappears for higher concentrations (Figure 2), as is found for drag reducing polymers in turbulent pipe flow. It was thought that the friction factor has been reduced by the similar mechanism as in turbulent pipe flow.

Before a definite conclusion is made that we are dealing with the drag reduction phenomenon, one must exclude possibilities of some viscous effects and effects introduced simply by a definition of Reynolds number. There were some suggestions (Fulford, 1964) that viscosity affects the mean film thickness (and hence the friction coefficient) more than it is accounted for by Reynolds number. In Figure 3 data on water and aqueous glycerol solutions (Tyler, 1971) are presented. For the range of viscosities studied, there was no additional effect of viscosity. For the case of dilute aqueous Carbopol solutions it might be concluded that an additional effect of apparent viscosity is not present either.

Reynolds number as given by (3) is defined through the use of the effective viscosity, that is, the viscosity which in tube flow makes Poiseuille's equation fit laminar flow conditions and applied to film flow. Another form of *Re* can be defined from similarity considerations of the film flow

$$Re_s = \frac{(4)\delta_0^n U_0^{2-n}\rho}{K} \tag{5}$$

where U_0 is the average film velocity.

Equation (5) when used with (1) also gives Equation (2).

Even for values of flow index which differ little from unity (the case of dilute Carbopol solutions), the difference in numerical values of the different forms of Reynolds number is not insignificant, as is seen in the Table 1. Still, this difference cannot account for the remarkable reduction in friction coefficient.

Doubts about the influence of the *Re* definition could be eliminated by considering the dependence of film thickness on the flow rate. For a nonwavy laminar flow, the following equation is readily obtained:

$$\delta_0 = \left[\frac{\left(\frac{2n+1}{n}\right)^n K Q^n}{\rho g} \right]^{\frac{1}{2n+1}} \tag{6}$$

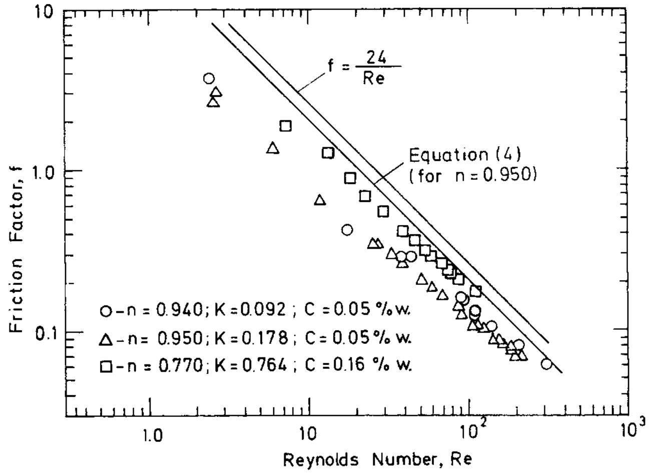

Fig. 2. Dependence of the friction factor on the Reynolds number for a falling film of aqueous Carbopol solutions.

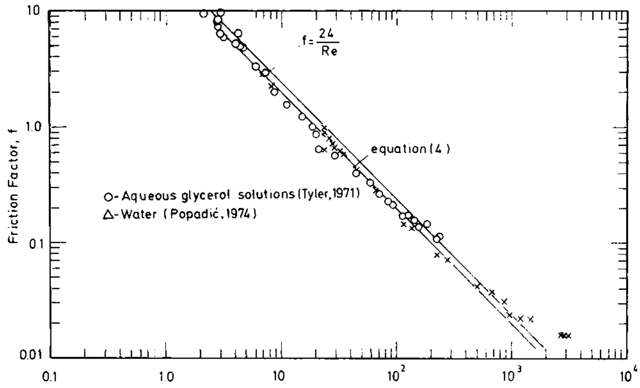

Fig. 3. Dependence of the friction factor on the Reynolds number for a falling film of Newtonian liquids.

For a wavy film similar to Equation (4), an approximative solution gives

$$\delta_0 = \left[\frac{\left(\frac{2n+1}{n}\right)^n K Q^n}{\rho g} \right]^{\frac{1}{2n+1}} \Phi^{\frac{n}{2n+1}} \quad (7)$$

There is no significant difference in numerical values obtained by Equations (6) and (7) (Popadic, 1974) so that Equation (6) was compared with the experimental data in Figure 4. The discrepancy is large, in particular for the smallest concentration. The liquid flows much faster than predicted. This was not found for Newtonian film flow as is reported many times (for example, Kapitza, 1948; Fulford, 1964; Tyler, 1971). It is evident from Figure 4 that for the higher polymer concentration the effect of drag reduction is smaller. The mean deviation between predicted and experimental values of the film thickness is 24.6% for $C = 0.05\%$ w and 11% for $C = 0.16\%$ w. Here the relative deviation is defined as

$$\frac{\text{predicted} - \text{experimental}}{\text{experimental}}.$$

This deviation is much larger when the friction coefficient is calculated by Equation (1) where it depends on the cube of the mean film thickness. The data points in Figure 4 are an average of 3 or 4 runs. The check on systematic error in the experiments was done by comparing data for water with the existing data, and an excellent agreement has been obtained. The random error was checked by running 30 runs with water for 3 different flow rates. The mean deviation was 0.82%.

In conclusion, we suggest that what is dealt here with is an unusual drag reduction in the flow field which is not turbulent, but the mechanism of reduction is similar to that of turbulent pipe flow.

The reduction in friction coefficient in the falling film of dilute polymer solutions might seem not to have a practical importance, but it shows that film heat transfer rates could be significantly decreased.

Table 1. Numeral Values of Different Forms of Reynolds Number

Liquid	Q (cm^3/cm s)	δ0 (cm)	Re	Re_s	f
$C = 0.05\%$ w	0.0336	0.0129	2.4	2.0	3.724
	0.2327	0.0261	17.3	14.2	4.297
$n = 0.940$	0.5015	0.0334	38.0	31.4	0.281
$K = 0.092 \frac{\text{dyne s}^n}{\text{cm}^2}$	0.5871	0.0372	44.6	36.7	0.293
	0.5810	0.0362	44.1	36.4	0.276
	1.1883	0.0482	91.9	75.1	0.156
$\rho = 1.008$ g/cm³					
	1.1611	0.0485	89.8	73.2	0.166
$\sigma = 70.4 \frac{\text{dyne}}{\text{cm}}$	1.4221	0.0508	110	90.3	0.127
	1.7994	0.0557	140	115	0.105
	2.3656	0.0622	186	151	0.084
	2.5805	0.0651	203	165	0.081
	3.8475	0.0773	306	247	0.061
	4.2055	0.0797	336	291	0.056
	0.0965	0.0203	7.1	5.9	1.762

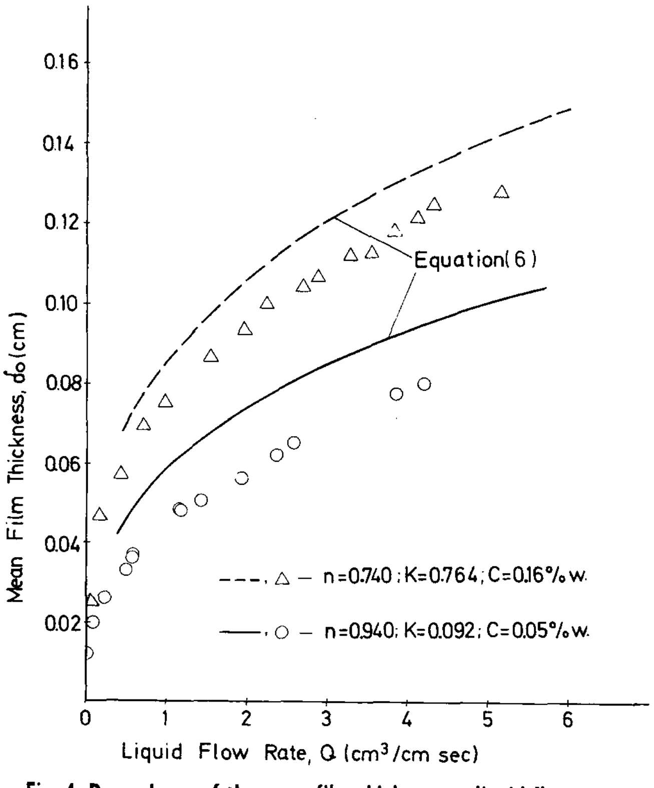

Fig. 4. Dependence of the mean film thickness on liquid flow rate.

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NOTATION

- C = concentration of Carbopol in water, % w
- f = friction coefficient
- g = gravity acceleration, cm/s²
- K = consistency factor, dyne s^n/cm^2
- n = flow index
- Q = volume flow rate per unit width of the plate, cm^3/cm s
- Re = Reynolds number defined by Equation (3)
- Re_s = Reynolds number defined by Equation (5)
- δ_0 = average film thickness, cm
- $\Phi(n)$ = measure of the deviation of film thickness in wavy from that in nonwavy laminar flow
- ρ = liquid density, g/cm³
- σ = surface tension, dyne/cm

LITERATURE CITED

Fulford, D. G., "The Flow of Liquids in Thin Films," in *Advances in Chem. Eng.*, Academic Press, New York (1964).

Gordon, R. I., C. Balakrishunan, and S. Pahwa, "Importance of Filament Formation in Turbulent Drag Reduction," *AIChE Symp. Ser., No. 130,* **69**, 33 (1973).

Kapitza, P. L., "Wave Flow of Thin Layers of a Viscous Fluid," *Zh. Exp. Teor. Fiz.*, **18**, 3 (1948).

Massot, C., F. Irani, and E. N. Lightfoot, "Modified Description of Wave Motion in a Falling Film," *AIChE J.*, **12**, 445 (1966).

Metzner, A. B., "Extensional Primary Field Approximations," *Rheol. Acta*, **10**, 434 (1971).

Popadic, V. O., "Pseudoplastic Thin Film Flow," Ph.D. dissertation, Univ. Kentucky, Lexington (1974).

Seyer, F. A., and A. B. Metzner, "Turbulence Phenomena in Drag-Reducing Systems," *AIChE J.*, **15**, 426 (1969).

Skelland, A. H. P., *Non-Newtonian Flow and Heat Transfer*, p. 75, Wiley, New York (1967).

Tyler, J. S., "Non-Newtonian Thin Film Flow," Ph.D. dissertation, Univ. Notre Dame, Indiana (1971).

Manuscript received November 11, 1974; revision received January 6 and accepted January 30, 1975.